Topic 0
Review of integration

Some derivatives:

$$\frac{d}{dx} x^n = n x^{n-1}$$

$$\frac{d}{dx} \sin(x) = \cos(x)$$

$$\frac{d}{dx} \cos(x) = \sin(x)$$

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

$$\frac{d}{dx} e^x = e^x$$

Some integrals:

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad \text{if } n \neq -1$$

$$\int \frac{1}{x} \, dx = \ln|x| + C$$

$$\int e^x \, dx = e^x + C$$

$$\int \sin(x) \, dx = -\cos(x) + C$$

$$\int \cos(x) \, dx = \sin(x) + C$$

Chain rule:

$$\left(f(g(x))\right)' = f'(g(x)) \cdot g'(x)$$

Ex:

$$\left(e^{5x}\right)' = e^{5x} \cdot 5 = 5e^{5x}$$

Ex:

$$\frac{d}{dx}\sin(2x^4+1) = \cos(2x^4+1) \cdot 8x^3$$
$$= 8x^3 \cos(2x^4+1)$$

Ex:

$$\left(\frac{5}{\sqrt{x^2+1}}\right)' = \left(5(x^2+1)^{-1/2}\right)'$$
$$= 5\left(-\tfrac{1}{2}\right)(x^2+1)^{-3/2} \cdot (2x)$$
$$= \frac{-5x}{(x^2+1)^{3/2}}$$

U-substitution:

$$\int f(g(x)) \cdot g'(x)\,dx = \int f(u)\,du$$

$u = g(x)$
$du = g'(x)\,dx$

Ex:

$$\int 2x e^{x^2}\,dx = \int e^u\,du = e^u + C$$

$$= e^{x^2} + C$$

$u = x^2$
$du = 2x\,dx$

Ex: $\int \sin(2x)\,dx = \frac{1}{2}\int \sin(u)\,du$

$$= -\frac{1}{2}\cos(u) + C$$

$$= -\frac{1}{2}\cos(2x) + C$$

$u = 2x$
$du = 2\,dx$
$\frac{1}{2}du = dx$

Ex: $\int \frac{1}{x(\ln(x))^2}\,dx$

$= \int \frac{1}{u^2}\,du = \int u^{-2}\,du$

$= \frac{u^{-1}}{-1} + C$

$= \frac{-1}{u} + C$

$= -\frac{1}{\ln(x)} + C$

$u = \ln(x)$

$du = \frac{1}{x}\,dx$

Integration by parts:

$$\int u\,dv = uv - \int v\,du$$

Ex:

$$\int x e^x dx = x e^x - \int e^x dx = x e^x - e^x + C$$

$u = x$ $\quad du = dx$

$dv = e^x dx$ $\quad v = e^x$
